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Global Equitable Domination Number of Some Wheel Related Graphs

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Abstract: A dominating set is called a global dominating set if it is a dominating set of a graph G and its complement $\overline{G}$. A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G) - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|d_G(u) - d_G(v)| \leq 1$. An equitable dominating set D of a graph G is a global equitable dominating set if it is also an equitable dominating set of the complement of G. The global equitable domination number $\gamma_g^e(G)$ of G is the minimum cardinality of a global equitable dominating set of G. In this paper, we investigate the global equitable domination number of some wheel related graphs.

Key Words: Global dominating set, equitable dominating set, Smarandachely equitable dominating set, global equitable dominating set, global equitable domination number.

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§1. Introduction

The study of domination in graphs is one of the fastest growing areas within graph theory. An excellent survey on the concept of domination and its related parameters can be found in the book by Haynes *et al.* [4] while some advanced topics on domination are explored in Haynes *et al.* [5]. The concept of domination has interesting applications in the study of social networks which motivated Prof. E. Sampathkumar to introduce the concept of equitable domination in graphs.

Secondly, let G be a graph of road network linking various locations. It is desirable to maintain the supply to these locations uninterruptedly by using the alternative links even if the original links get disturbed. Then the problem of finding the minimum number of supplying stations needed to accomplish this task is equivalent to find the global domination number. The concept of global domination was introduced by Sampathkumar [9].

Many domination models are introduced by combining two different domination parameters. Independent domination, global domination, equitable domination, connected domination are among worth to mention. Motivated through the concepts of global domination and equitable domination, a new concept of global equitable domination was conceived by Basavanagoud

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and Teli [2] and formalized by Vaidya and Pandit [14]. In the present paper, we obtain the global equitable domination number of some wheel related graphs.

Throughout the paper, a graph $G = (V(G), E(G))$ we mean a finite and undirected graph without loops and multiple edges. The set $D \subseteq V(G)$ of vertices in a graph G is called a dominating set if every vertex $v \in V(G)$ is either an element of D or is adjacent to an element of D. The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$.

The complement $\overline{G}$ of G is the graph with vertex set $V(G)$ in which two vertices are adjacent in $\overline{G}$ if they are not adjacent in G.

For a vertex $v \in V(G)$, the open neighborhood of v, denoted by $N(v)$, is $\{u \in V(G) : uv \in E(G)\}$. We denote the degree of a vertex v in G by $d_G(v)$. A vertex of degree one is called a pendant vertex and a vertex which is not the end of any edge is called an isolated vertex. An edge e of a graph G is said to be incident with the vertex v if v is an end vertex of e. An edge incident with a pendant vertex is called a pendant edge.

A set $D \subseteq V(G)$ is called a global dominating set of G if D is a dominating set of both G and $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a global dominating set in G. Many researchers have explored this concept. For example, Gangadharappa and Desai [3] have discussed the global domination in graphs of small diameters. Vaidya and Pandit [12, 13] have investigated the global domination number of the larger graphs obtained by some graph operations on a given graph while Kulli and Janakiram [6] have introduced the concept of total global dominating sets.

A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G) - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|d_G(u) - d_G(v)| \leq 1$, otherwise, a Smarandachely equitable dominating set, i.e., $|d_G(u) - d_G(v)| \geq 2$ for each edge $uv \in E(G)$ with $u \in D$ and $v \in V(G) - D$. The minimum cardinality of such a dominating set is called the equitable domination number of G which is denoted by $\gamma^e(G)$. Swaminathan and Dharmalingam [11] have studied the equitable domination in graphs and characterized the minimal equitable dominating sets. Sivakumar *et al.* [10] have discussed the connected equitable domination in graphs while Murugan and Emmanuel [7] have identified the inter relationship among domination, equitable domination and independent domination in graphs. Revathi and Harinarayanan [8] have studied the equitable domination in fuzzy graphs while Basavanagoud *et al.* [1] have studied the equitable total domination in graphs.

A vertex $v \in V(G)$ is equitably adjacent with a vertex $u \in V(G)$ if $|d_G(u) - d_G(v)| \leq 1$ and $uv \in E(G)$. A vertex $u \in V(G)$ is called an equitable isolate if $|d_G(u) - d_G(v)| \geq 2$ for all $v \in N(u)$. Analogous to the characteristic of an isolated vertex in a dominating set, an equitable isolate must belong to any equitable dominating set of G. Clearly, the isolated vertices are the equitable isolates. Hence, $I_s \subseteq I_e \subseteq D$ for every equitable dominating set D where I_s and I_e denote the sets of all isolated vertices and all equitable isolates of G respectively.

A subset D of $V(G)$ is called a global equitable dominating set of G if D is an equitable dominating set of both G and $\overline{G}$. The minimum cardinality of a global equitable dominating set of G is called the global equitable domination number of G and it is denoted by $\gamma_g^e(G)$.

Since at least two vertices are required to equitably dominate both G and $\overline{G}$, we have

$2 \leq \gamma_g^e(G) \leq n$ for every graph of order $n > 1$. Both of these bounds are sharp. In particular, the equality of the lower bound is attained by P_n $(2 \leq n \leq 6)$ and $K_{r,s}$ $(|r-s| \leq 1)$ while the upper bound is achieved by K_n, $K_{1,p}$ and $K_{r,s}$ $(|r-s| \geq 2)$.

The wheel W_n is defined to be the join $C_{n-1} + K_1$ where $n \geq 4$. The vertex corresponding to K_1 is known as the apex vertex and the vertices corresponding to cycle C_{n-1} are known as the rim vertices. For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n and $\lfloor n \rfloor$ denotes the greatest integer not greater than n.

For notations and graph theoretic terminology not defined herein, we refer the readers to West [15] while the terms related to the concept of domination are used in the sense of Haynes *et al.* [4].

§2. Main Results

Definition 2.1 *The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each of its rim vertices.*

Proposition 2.2 ([2])

(*i*) *For the path P_n $(n \geq 4)$, $\gamma_g^e(P_n) = \lceil \frac{n}{3} \rceil$;*

(*ii*) *For the cycle C_n,* $\gamma_g^e(C_n) = \begin{cases} 3 & \text{if } n = 3, 5 \\ \lceil \frac{n}{3} \rceil & \text{otherwise.} \end{cases}$

Theorem 2.3 *For the helm,* $\gamma_g^e(H_n) = \begin{cases} 7 & \text{if } n = 4 \\ n+2 & \text{if } n = 5, 6 \\ \lceil \frac{4n-1}{3} \rceil & \text{otherwise.} \end{cases}$

Proof Let $v_1, v_2, \cdots, v_{n-1}$ be the rim vertices of wheel W_n and let c denotes the apex vertex of the helm H_n. Let $u_1, u_2, \ldots, u_{n-1}$ be the pendant vertices of H_n. Then, $|V(H_n)| = 2n-1$ and $|E(H_n)| = 3(n-1)$.

Case 1. $n = 4$

For $n = 4$, the pendant vertices of H_n are equitable isolates in H_n while the remaining vertices of H_n are equitable isolates in $\overline{H_n}$. Hence, the vertex set of H_n is the only global equitable dominating set of H_n implying that $\gamma_g^e(H_n) = |V(H_n)| = 7$.

Case 2. $n = 5, 6$

Since the $n-1$ pendant vertices of H_n are equitable isolates in H_n and the apex vertex c is an equitable isolate in $\overline{H_n}$, it follows that every global equitable dominating set of H_n must contain these vertices. Now, these vertices equitably dominate all the vertices of H_n but do not equitably dominate all the vertices of $\overline{H_n}$. Moreover, any two adjacent rim vertices of W_n can equitably dominate the remaining vertices of $\overline{H_n}$. Hence, every global equitable dominating set of H_n must contain at least $n+2$ vertices of H_n. Therefore, $\gamma_g^e(H_n) = n+2$.

Case 3. $n \geq 7$

In this case, the apex vertex is an equitable isolate in H_n as well as in $\overline{H_n}$ while the $n-1$ pendant vertices of H_n are equitable isolates in H_n only. Therefore, these vertices must belong to every global equitable dominating set of H_n. Now, the remaining vertices induce a cycle C_{n-1} and by Proposition 2.2, $\gamma_g^e(C_{n-1}) = \lceil \frac{n-1}{3} \rceil$. Therefore, $\gamma_g^e(H_n) = n-1+1+\lceil \frac{n-1}{3} \rceil = \lceil \frac{4n-1}{3} \rceil$.

Thus, we have proved that

$$\gamma_g^e(H_n) = \begin{cases} 7 & \text{if } n=4 \\ n+2 & \text{if } n=5,6 \\ \lceil \frac{4n-1}{3} \rceil & \text{otherwise.} \end{cases}$$

□

Definition 2.4 *The flower graph Fl_n is the graph obtained from the helm H_n by joining each pendant vertex to the apex vertex of the helm H_n.*

Theorem 2.5 *For the flower graph,*

$$\gamma_g^e(Fl_n) = \begin{cases} n+3 & \text{if } n=4,6 \\ \lceil \frac{4n-1}{3} \rceil & \text{otherwise.} \end{cases}$$

Proof Let $v_1, v_2, \cdots, v_{n-1}$ be the rim vertices of wheel W_n and let $u_1, u_2, \cdots, u_{n-1}$ be the pendant vertices of the helm H_n. Let c denotes the apex vertex of Fl_n. Then $|V(Fl_n)| = 2n-1$. Here, $d_G(v_i) = 4$, $d_G(u_i) = 2$ for $1 \leq i \leq n-1$ and $d_G(c) = 2(n-1)$ where $G = Fl_n$.

Now, the vertex c is an equitable isolate in G as well as in $\overline{G}$. Therefore, every global equitable dominating set of G must contain c. Moreover, the vertices $u_1, u_2, \ldots, u_{n-1}$ being the equitable isolates in G, must belong to every global equitable dominating set of G. Now, the remaining vertices $v_1, v_2, \ldots, v_{n-1}$ in G induce a cycle C_{n-1} and by Proposition 2.2,

$$\gamma_g^e(C_n) = \begin{cases} 3 & \text{if } n=3,5 \\ \lceil \frac{n}{3} \rceil & \text{otherwise.} \end{cases}$$

Hence,

$$\begin{aligned} \gamma_g^e(Fl_n) &= \gamma_g^e(C_{n-1}) + (n-1) + 1 \\ &= \gamma_g^e(C_{n-1}) + n \\ &= \begin{cases} n+3 & \text{if } n=4,6 \\ \lceil \frac{n-1}{3} \rceil + n & \text{otherwise.} \end{cases} \end{aligned}$$

Thus,

$$\gamma_g^e(Fl_n) = \begin{cases} n+3 & \text{if } n=4,6 \\ \lceil \frac{4n-1}{3} \rceil & \text{otherwise.} \end{cases}$$

□

Definition 2.6 *The sunflower graph Sf_n is the resultant graph obtained from the flower graph by attaching $(n-1)$ pendant edges to the apex vertex of wheel W_n.*

Theorem 2.7 *For the sunflower graph, $\gamma_g^e(Sf_n) = 3n-2$.*

Proof Let c denotes the apex vertex of wheel W_n and let $v_1, v_2, \cdots, v_{n-1}$ be the rim vertices of W_n. Let $u_1, u_2, \cdots, u_{n-1}$ be the vertices of degree 2 in Sf_n and let $x_1, x_2, \ldots, x_{n-1}$ be the pendant vertices of Sf_n. Then, $|V(Sf_n)| = 3n-2$.

Now, c is the equitable isolate in both Sf_n and $\overline{Sf_n}$. Moreover, the vertices $u_1, u_2, \cdots, u_{n-1}$, $x_1, x_2, \cdots, x_{n-1}$ are equitable isolates in Sf_n while the vertices $v_1, v_2, \cdots, v_{n-1}$ are equitable isolates in $\overline{Sf_n}$. Since an equitable isolate must belong to every equitable dominating set, it follows that the vertex set $V(Sf_n)$ is the only global equitable dominating set of Sf_n. Therefore, $\gamma_g^e(Sf_n) = |V(Sf_n)| = 3n-2$. □

Definition 2.8 *The closed helm CH_n is the graph obtained from a helm by joining each pendant vertex to form a cycle.*

Theorem 2.9 *For the closed helm CH_n $(n > 5)$,*

$$\gamma_g^e(CH_n) = \begin{cases} \left\lfloor \frac{n+2}{2} \right\rfloor & \text{if n} \equiv 1 \pmod 4 \\ \left\lceil \frac{n+2}{2} \right\rceil & \text{if n} \equiv 0,\ 2\ \mathit{or}\ 3 \pmod 4. \end{cases}$$

Proof Let $v_1, v_2, \cdots, v_{n-1}$ be the vertices of degree 4 and let $u_1, u_2, \cdots, u_{n-1}$ be the vertices of degree 3 in $G = CH_n$. Let c denotes the apex vertex of CH_n. Then the closed helm CH_n has $2n-1$ vertices. The vertex c is an equitable isolate in CH_n and the remaining vertices which are adjacent in CH_n, are also equitably adjacent in CH_n. The vertex c being an equitable isolate, must belong to every global equitable dominating set of CH_n. Hence, we construct a vertex set $D \subset V(CH_n)$ as follows:

$$D = \{c, v_{4i+1}, u_{4j+3}\},$$

where $0 \le i \le \left\lfloor \frac{n-2}{4} \right\rfloor$ and $0 \le j \le \left\lfloor \frac{n}{4} \right\rfloor$ with

$$|D| = \begin{cases} \left\lfloor \frac{n+2}{2} \right\rfloor & \text{if n} \equiv 1 \pmod 4 \\ \left\lceil \frac{n+2}{2} \right\rceil & \text{if n} \equiv 0,\ 2\ \mathit{or}\ 3 \pmod 4. \end{cases}$$

Now, $d_G(v_{4i+1}) = 4$, $d_G(u_{4j+3}) = 3$ and $d_{\overline{G}}(u_{4j+3}) - d_{\overline{G}}(v_{4j+1}) = 1$. Then for every $v \in V(G) - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|d_G(u) - d_G(v)| \le 1$. Moreover, for every $v' \in V(G) - D$ there exists a vertex $u' \in D$ such that $u'v' \in E(\overline{G})$ and $|d_{\overline{G}}(u') - d_{\overline{G}}(v')| \le 1$. Hence, the set D is an equitable dominating set of G as well as of $\overline{G}$. Therefore, D is a global equitable dominating set of G. Moreover, from the adjacency nature of the vertices of G, one can observe that the set D is of minimum cardinality.

Thus, the set D is a global equitable dominating set of $G = CH_n$ $(n > 5)$ with minimum cardinality implying that

$$\gamma_g^e(CH_n) = \begin{cases} \lfloor \frac{n+2}{2} \rfloor & \text{if n} \equiv 1 \,(\text{mod}\,4) \\ \lceil \frac{n+2}{2} \rceil & \text{if n} \equiv 0,\ 2\ \textit{or}\ 3\ (\text{mod}\,4). \end{cases}$$

□

Remark 2.10 For $n = 4, 5$, at least two vertices are required to equitably dominate all the vertices of CH_n as well as of $\overline{CH_n}$. Therefore, $\gamma_g^e(CH_n) = 2$ for $n = 4, 5$.

Definition 2.11 *A web graph is the graph obtained by joining the pendant vertices of a helm to form a cycle and then adding a single pendant edge to each vertex of this outer cycle. We denote the web graph by Wb_n.*

Theorem 2.12 *For the web graph Wb_n $(n > 6)$,*

$$\gamma_g^e(Wb_n) = \begin{cases} \lfloor \frac{3n}{2} \rfloor & \text{if n} \equiv 1 \,(\text{mod}\,4) \\ \lceil \frac{3n}{2} \rceil & \text{if n} \equiv 0,\ 2\ \textit{or}\ 3\ (\text{mod}\,4). \end{cases}$$

Proof Let c denotes the apex vertex of web graph Wb_n. Let $v_1, v_2, \cdots, v_{n-1}$ and $u_1, u_2, \cdots, u_{n-1}$ be the vertices of inner cycle and outer cycle of Wb_n respectively. Let $x_1, x_2, \cdots, x_{n-1}$ denote the pendant vertices of Wb_n.

Since the apex vertex c and the $n - 1$ pendant vertices are the equitable isolates in Wb_n as well as in $\overline{Wb_n}$ for $n > 6$, it follows that these vertices must belong to every global equitable dominating set of Wb_n. Moreover, the vertices except the pendant vertices induce the closed helm CH_n and by Theorem 2.9, we have

$$\gamma_g^e(CH_n) = \begin{cases} \lfloor \frac{n+2}{2} \rfloor & \text{if n} \equiv 1 \,(\text{mod}\,4) \\ \lceil \frac{n+2}{2} \rceil & \text{if n} \equiv 0,\ 2\ \textit{or}\ 3\ (\text{mod}\,4). \end{cases}$$

Hence,

$$\gamma_g^e(Wb_n) = \gamma_g^e(CH_n) + (n-1)$$

$$= \begin{cases} \lfloor \frac{n+2}{2} \rfloor + (n-1) & \text{if n} \equiv 1 \,(\text{mod}\,4) \\ \lceil \frac{n+2}{2} \rceil + (n-1) & \text{if n} \equiv 0,\ 2\ \textit{or}\ 3\ (\text{mod}\,4). \end{cases}$$

Thus, for $n > 6$,

$$\gamma_g^e(Wb_n) = \begin{cases} \lfloor \frac{3n}{2} \rfloor & \text{if n} \equiv 1 \,(\text{mod}\,4) \\ \lceil \frac{3n}{2} \rceil & \text{if n} \equiv 0,\ 2\ \textit{or}\ 3\ (\text{mod}\,4). \end{cases}$$

□

Remark 2.13 (i) For $n = 4, 5$, the $n - 1$ pendant vertices are the equitable isolates in Wb_n

as well as in $\overline{Wb_n}$ and the remaining vertices induce the closed helm CH_n. Thus, $\gamma_g^e(Wb_n) = \gamma_g^e(CH_n) + (n-1)$ implying that $\gamma_g^e(Wb_4) = 5$ and $\gamma_g^e(Wb_5) = 6$.

(ii) For $n = 6$, the apex vertex is not an equitable isolate in Wb_6 as well as in $\overline{Wb_6}$ while the pendant vertices are the equitable isolates in both Wb_6 and $\overline{Wb_6}$. Hence, $\gamma_g^e(Wb_6) = 8$.

Definition 2.14 *A gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the $(n-1)$ - cycle of W_n.*

Theorem 2.15 *For the gear graph,*

$$\gamma_g^e(G_n) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{if } n = 4, 5 \\ \lceil \frac{2n+1}{3} \rceil & \text{otherwise.} \end{cases}$$

Proof Let c denotes the apex vertex of wheel W_n and let $v_1, v_2, \cdots, v_{n-1}$ be the rim vertices of W_n. To obtain the gear graph G_n, subdivide each rim edge of wheel by the vertices $u_1, u_2, \cdots, u_{n-1}$ where each u_i is added between v_i and v_{i+1} for $i = 1, 2, \cdots, n-2$ and u_{n-1} is added between v_1 and v_{n-1}. Then $|V(G_n)| = 2n - 1$ and $|E(G_n)| = 3(n-1)$. The graph G_n contains the outer cycle $C_{2(n-1)}$.

For $n = 4, 5$, the sets $D = \{v_1, u_3\}$ and $D = \{c, v_1, v_3\}$ are clearly the global equitable dominating sets of G_4 and G_5 respectively with minimum cardinality. Therefore, $\gamma_g^e(G_n) = \lceil \frac{n}{2} \rceil$ for $n = 4, 5$.

For $n > 5$, since the vertex c is the equitable isolate in G_n as well as in $\overline{G_n}$, it must belong to every global equitable dominating set of G_n. Moreover, the vertices other than c induce a cycle $C_{2(n-1)}$. Furthermore, $V(G_n) = V(C_{2(n-1)}) \bigcup \{c\}$ and by Proposition 2.2, $\gamma_g^e(C_n) = \lceil \frac{n}{3} \rceil$ for $n > 5$. This implies that $\gamma_g^e(G_n) = \gamma_g^e(C_{2(n-1)}) + 1 = \left\lceil \frac{2(n-1)}{3} \right\rceil + 1 = \lceil \frac{2n+1}{3} \rceil$. Hence, we have proved that

$$\gamma_g^e(G_n) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{if } n = 4, 6 \\ \lceil \frac{2n+1}{3} \rceil & \text{otherwise.} \end{cases}$$

□

Definition 2.16 *The splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.*

Theorem 2.17 *For the splitting graph of wheel W_n $(n > 7)$,*

$$\gamma_g^e(S'(W_n)) = \begin{cases} \lceil \frac{4n+3}{3} \rceil & \text{if n} \equiv 0 \textit{ or } 2 \pmod 3 \\ \lfloor \frac{4n+3}{3} \rfloor & \text{if n} \equiv 1 \pmod 3. \end{cases}$$

Proof Let $v_1, v_2, \cdots, v_{n-1}$ be the rim vertices of wheel W_n and let c denotes the apex vertex of W_n. Let $c', v'_1, v'_2, \cdots, v'_{n-1}$ be the added vertices corresponding to the vertices $c, v_1, v_2, \cdots, v_{n-1}$ of W_n to obtain $G = S'(W_n)$. Then $|V(G)| = 2n$.

For $n = 8$, the vertices c and c' are equitable isolates in $\overline{G}$ and $c, v'_1, v'_2, \cdots, v'_7$ are equitable isolates in G. For $n > 8$, the vertices c and c' are equitable isolates in both G and $\overline{G}$ while the vertices $v'_1, v'_2, \cdots, v'_{n-1}$ are equitable isolates in G. Since an equitable isolate must belong to every equitable dominating set of G, the vertices $c, c', v'_1, v'_2, \cdots, v'_{n-1}$ being equitable isolates, must belong to every global equitable dominating set of G. Now, the remaining vertices $v_1, v_2, \cdots, v_{n-1}$ of G induce a cycle C_{n-1} and by Proposition 2.2, $\gamma_g^e(C_n) = \lceil \frac{n}{3} \rceil$ for $n > 5$. This implies that $\gamma_g^e(G) = \gamma_g^e(C_{n-1}) + n + 1 = \lceil \frac{n-1}{3} \rceil + n + 1$. Thus, for $n > 7$,

$$\gamma_g^e(S'(W_n)) = \begin{cases} \lceil \frac{4n+3}{3} \rceil & \text{if n} \equiv 0 \ or\ 2 \pmod 3) \\ \lfloor \frac{4n+3}{3} \rfloor & \text{if n} \equiv 1 \pmod 3). \end{cases}$$

□

Remark 2.18 For $4 \leq n \leq 7$, the apex vertex and all the duplicated vertices are the equitable isolates either in $S'(W_n)$ or in $\overline{S'(W_n)}$ and by Proposition 2.2,

$$\gamma_g^e(C_n) = \begin{cases} 3 & \text{if } n = 3, 5 \\ \lceil \frac{n}{3} \rceil & \text{otherwise.} \end{cases}$$

Hence,

$$\gamma_g^e(S'(W_n)) = \begin{cases} 8 & \text{if } n = 4, 5 \\ 10 & \text{if } n = 6, 7. \end{cases}$$

§3. Concluding Remarks

The concept of global equitable domination is a variant of global domination and equitable domination. We obtain the exact values of global equitable domination number of the helm H_n, the flower graph Fl_n, the sunflower graph Sf_n, the closed helm CH_n, the web graph Wb_n, the gear graph G_n and the splitting graph of wheel $S'(W_n)$.

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